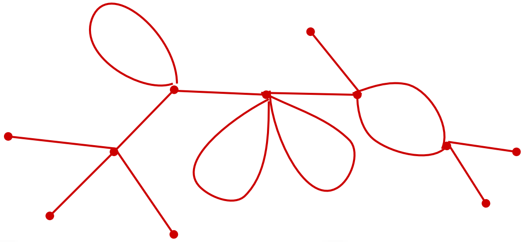
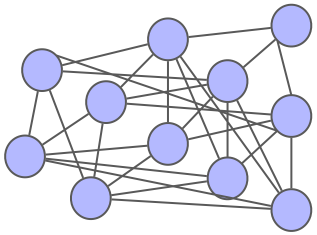
Neural Network Field Theories





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String Phenomenology '22 | Parallel Talks

Based on arxiv:2008.08601, 2106.00694, & 22xx.xxxxx (w/ Jim Halverson, Matt Schwartz, Mehmet Demirtas, Keegan Stoner)

Wait, Why Neural Networks?

Neural Networks (NN) are the backbones of breakthroughs in Machine Learning.

Given input x, Neural Nets generate output function $\phi(x)$.

Training / Learning helps a Neural Net approximate any desired function - Supervised Learning, Reinforcement Learning, Generative Models, Natural Language Processing.

Some examples - AlphaGo for chess, GPT-3 for language models, DALL-E for images.

Also used in string theory, knot theory, EFT model building!

[Halverson, Long 2020]
[Halverson, Nelson, Ruehle 2019]
[Larfos, Lukas, Ruehle, Schneider 2022]

image source: google

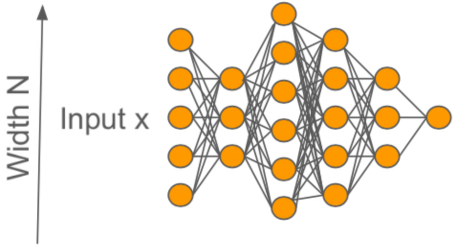
Neural Networks As Field Theories?

A single Neural Network (NN) output is a function composed of parameters and an architecture. E.g.

$$\phi(x) = \frac{1}{\sqrt{N}} \sum_{i=1}^{N} h_i(x) \quad \Longleftrightarrow \quad Z[\phi] = \int D\phi \, e^{-S[\phi]}$$

Ensembles of NN outputs are functional distributions. So are field theories.

- Lim $N \to \infty$ and independent neurons h_i: free scalar fields.
- Close to Lim $N \to \infty$ and / or small correlations in h_i: weakly coupled scalar fields.



Far from Lim $N \to \infty$ and / or strong correlations in h_i : NN ensembles behave as non-perturbative non-Lagrangian field theories.

Outline

- Free NN Field Theories
- Weakly Coupled NN Field Theories
- Non-perturbative Non-Lagrangian NN Field Theories
    - Correlation Functions
    - Partition Function
    - Symmetries

Free NN Field Theories

$$\phi(x) = \frac{1}{\sqrt{N}} \sum_{i=1}^{N} h_i(x)$$

Central Limit Theorem (CLT): A sum over infinite independent variables is a draw from a Gaussian process.

⁕ Free scalar field theory is a Gaussian process too!

NN output $\phi(x)$ behaves as free scalar field on Euclidean background. Input x behaves as Euclidean space-time.

$$S[\phi] = \frac{1}{2} \int d^d x d^d y \, \phi(x) \left(G^{(2)}(x, y) \right)^{-1} \phi(y)$$

Analytic continuation to Minkowskian spacetime — when correlation functions satisfy Osterwalder-Schrader axioms ⟹ NN ensembles behave as quantum fields.

Weakly Coupled NN Field Theories

$$\phi(x) = \frac{1}{\sqrt{N}} \sum_{i=1}^{N} h_i(x)$$

Finite large N and / or small correlations in neurons h_i softly break the Central Limit Theorem.

Non-Gaussianities lead to weakly coupled interaction terms in NN action.
E.g.

$$S[\phi] = \underbrace{\frac{1}{2} \int d^d x d^d y \, \phi(x) \left(G^{(2)}(x,y)\right)^{-1} \phi(y)}_{S_{free}[\phi]} + \underbrace{\int d^d x d^d y \sum_{i=0}^{3} \lambda_i(x,y) \, \phi^i(x) \, \phi^{4-i}(y)}_{\Delta S[\phi]}$$

Experimental correlators of NN ensembles match predictions by Feynman diagrams.

Non-perturbative Non-Lagrangian NNFT

Far from $\lim N \to \infty$ and / or highly correlated neurons h_i: NN field theory is no longer perturbative, action is *unknown*.

There exists a dual framework in terms of neurons.

This duality lets us study correlators, symmetries etc in terms of neuron distributions.

Field Space

$$\phi(x)$$

$$G^{(n)}(x_1, \cdots, x_n) = \frac{1}{Z} \int D\phi \, e^{-S[\phi]} \, \phi(x_1) \cdots \phi(x_n)$$

Neuron Space

$$\frac{1}{\sqrt{N}} \sum_{i=1}^{N} h_i(x)$$

$$G^{(n)}(x_1, \cdots, x_n) = \frac{1}{N^{n/2}} \sum_{i_1, \cdots, i_n} \int Dh \, P(h) \, h_{i_1}(x_1) \cdots h_{i_n}(x_n)$$

Correlation Functions

We can study NNFT cumulants / connected correlators using neuron distributions.

Cumulant Generating Functional (CGF) technique:

$$W[J] := \sum_{r=1}^{\infty} \frac{J^r \, G_{con}^{(r),\phi}(x_1, \cdots, x_r)}{r!} = \log\left[\int Dh\, P(h)\, e^{\frac{1}{\sqrt{N}} \sum_{i=1}^{N} \int d^d x\, h_i(x) J(x)} \right]$$

Field space

Neuron space

Independent & identical neurons : $G_{con}^{(r),\phi}(x_1, \cdots, x_r) = \frac{G_{con}^{(r),h_i}(x_1, \cdots, x_r)}{N^{\frac{r}{2}-1}}$

Correlated neurons : each $G_{con}^{(r),\phi}(x_1, \cdots, x_r)$ receives contributions from multiple cumulants of all neurons.

Partition Function

Edgeworth expansion technique:

$$Z_\phi[J] = \int DhD\phi\, P(h)\, e^{-\int d^dx\, J(x)\phi(x)}\, \delta\left[\phi(x) - \frac{1}{\sqrt{N}}\sum_{i=1}^{N} h_i(x)\right]$$

This expresses NN field theory partition functions in non-perturbative non-Lagrangian regime as series expansions around free NNFT PDF.

$$Z_\phi[0] = \int D\phi\, e^{\sum_{r=3}^{\infty} \frac{(-i)^r}{r!} \int dx_1 \cdots dx_r\, G^{(r),\phi}_{con,(x_1,\cdots,x_r)} \frac{\partial}{\partial\phi(x_1)} \cdots \frac{\partial}{\partial\phi(x_r)}}\, e^{-\frac{1}{2}\int d^dxd^dy\, \phi(x)\left(G^{(2)}(x,y)\right)^{-1}\phi(y)}$$

$P_{free}[\phi]$

◆ Symmetries

Symmetries of neuron distributions

→ Symmetries of NN correlators in neuron space

↕

Symmetries of NN field theory correlators

↙

Symmetries of NN field theory action

Symmetries of NN inputs	⟷	Symmetries of space-time
Symmetries of NN outputs	⟷	Symmetries of quantum fields

Future Goals

- Construct known quantum field theories using Neural Network ensembles.
- Based on above, we can use different Neural Net architectures to introduce more than one fields in the theory.

Thank you!

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Questions?